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RESEARCH MEMORANDUM

OUTPUT CONSISTENCY AND WEAK OUTPUT CONSISTENCY FOR CONTINUOUS-TIME IMPLICIT SYSTEMS

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OUTPUT CONSISTENCY AND WEAK OUTPUT CONSISTENCY FOR CONTINUOUS-TIME IMPLICIT SYSTEMS

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ABSTRACT

In a recent paper, the concept of consistency of initial conditions for general continuous-time *implicit* systems was recovered as a special case of so-called *weak* consistency, and it was demonstrated that "global" weak consistency is equivalent to impulse controllability. In this paper, both concepts are generalized for continuous-time implicit systems with a given output, and we derive necessary and sufficient conditions for "global" *output consistency* as well as "global" *weak output consistency*. Elsewhere these results will be used for a complete treatment of consistency and relaxations of consistency for arbitrary higher order implicit systems. Moreover, our conditions reduce to the known ones for (weak) *state* consistency if the output and state variable are the same.

KEYWORDS

Continuous-time implicit systems, state consistency, weak state consistency, output consistency, weak output consistency.

1 Introduction.

Recently [1], issues such as solvability and consistency were investigated in depth for linear systems of the form

$$E\dot{x}(t) = Ax(t) + Bu(t), \tag{1}$$

with $E, A \in R^{l\times n}, B \in R^{l\times m}, u(t) \in R^m, x(t) \in R^n$ for all $t \in R^+ = [0,\infty)$ and $x_0 := x(0^-) \in R^n$, arbitrary. Following [2], a point $x_0 \in R^n$ is called *consistent* if (1) has a classical solution $x : R^+ \to R^n$ with $x(0^+) = x_0$, and *inconsistent* if this is not the case. However, by redefining (1) in terms of *distributions* [3], it can be seen that inconsistent points may give rise to *impulsive* solutions of (1) [4] - [5]. A simple distributional framework for (1) is given in [1]. This setup is based on the commutative algebra (over R) of *impulsive-smooth* distributions $\mathcal{C}_{imp}$ [6, Definition 3.1], [7] (here, convolution $*$ of distributions plays the role of multiplication). An impulsive-smooth distribution is any linear combination of an *impulse* and a *smooth* distribution. An impulse or impulsive distribution is any linear combination of the unit element in $\mathcal{C}_{imp}$, the Dirac distribution δ, and its (distributional) derivatives $\delta^{(i)}(i \geq 1)$. A smooth distribution corresponds to a function that is zero on $(-\infty, 0)$ and arbitrarily often differentiable on R^+(in the usual sense). Let $\mathcal{C}_{sm}$, $\mathcal{C}_{p-imp}$ denote the subalgebras of smooth distributions and impulses, respectively. The distributional derivative $u^{(1)}$ of $u \in \mathcal{C}_{imp}$ equals $\delta^{(1)} * u$. If $u = u_1 + u_2$ with $u_1 \in \mathcal{C}_{p-imp}$ and $u_2 \in \mathcal{C}_{sm}$, then $u(0^+) := u_2(0^+) := \lim_{t \downarrow 0} u_2(t)$. If $u \in \mathcal{C}_{sm}$, then $u^{(1)} = \dot{u} + u(0^+)\delta$, where $\dot{u}$ denotes the ordinary derivative of u. It holds that $\delta^{(i)} = \delta^{(i-1)} * \delta^{(1)}(i \geq 1)$ with $\delta^{(0)} := \delta$, and by defining $\delta^{(-1)} := H$, the Heaviside "unit step" distribution, and $\delta^{(-j)} := \delta^{-(j-1)} * \delta^{(-1)}(j \geq 1)$, we establish that $\delta^{(i+j)} = \delta^{(i)} * \delta^{(j)}(i, j \in Z)$ and thus the inverse (w.r.t to convolution) of $\delta^{(i)}, (\delta^{(i)})^{-1}$, equals $\delta^{(-i)}(i \in Z), \delta^{-1} = \delta$.

Then, instead of (1), we consider the distributional equation

$$\delta^{(1)} * Ex = Ax + Bu + Ex_0\delta, \tag{2}$$

together with, for every pair $(x_0, u) \in R^n \times \mathcal{C}^m_{imp}$ (the m-vector version of $\mathcal{C}_{imp}$), the *solution set* [1, (2.2b)]

$$S(x_0, u) = \{x \in \mathcal{C}^n_{imp} \mid [E\delta^{(1)} - A\delta] * x = Bu + Ex_0\delta\}. \tag{3}$$

Definition 1.1[1, Definition 4.1].

A point $x_0 \in R^n$ is called consistent if

$$\exists\, u \in \mathcal{C}^m_{sm} \exists\, x \in S(x_0, u) \cap \mathcal{C}^n_{sm} : x(0^+) = x_0.$$

A point $x_0 \in R^n$ is called *weakly* consistent if

$$\exists\, u \in \mathcal{C}^m_{sm} : S(x_0, u) \cap \mathcal{C}^n_{sm} \neq \emptyset.$$

Proposition 1.2 [1, Theorem 4.5].

Assume that $[EAB]$ is of full row rank. Then every $x_0 \in R^n$ is consistent if and only if $im(E) + im(B) = R^l$. Every $x_0 \in R^n$ is weakly consistent if and only if $im(E) + im(B) + A[ker(E)] = R^l$.

Now it is the purpose of this paper to generalize the concepts in Definition 1.1 and the results in Proposition 1.2 for systems (2) with *output* equation

$$y = Cx, \tag{4}$$

where $C \in R^{r \times n}$. It will be shown elsewhere that these generalizations are of great value for a thorough treatment of solvability, consistency and relaxations of consistency for *higher order* linear systems on R^+ of the form

$$A_k[d^k x/dt^k] + A_{k-1}[d^{k-1}x/dt^{k-1}] + \cdots + A_1\dot{x}(t) + A_0 x(t) = Bu(t), \tag{5}$$

with, for all $i = 0, 1, \ldots, k$, $A_i \in R^{l \times n}$, *arbitrary*, and $B \in R^{l \times m}$.

In the sequel we will frequently use some trivial observations.

Lemma 1.3.

Let $\mathcal{L} \subset R^n$ and $M \in R^{l \times n}$. Then $M[M^{-1}(\mathcal{L})] = \mathcal{L} \cap im(M)$ and $M^{-1}[M(\mathcal{L})] = \mathcal{L} + ker(M)$. If $\mathcal{L}_{1,2} \subset R^n$ and $ker(M) \subset \mathcal{L}_1$ and/or $ker(M) \subset \mathcal{L}_2$, then $M(\mathcal{L}_1 \cap \mathcal{L}_2) = M(\mathcal{L}_1) \cap M(\mathcal{L}_2)$. If $\mathcal{L}_{1,2,3} \subset R^n$ and $\mathcal{L}_2 \subset \mathcal{L}_3$, then $(\mathcal{L}_1 + \mathcal{L}_2) \cap \mathcal{L}_3 = \mathcal{L}_2 + [\mathcal{L}_1 \cap \mathcal{L}_3]$ (*modular* rule).

2 Output consistency and weak output consistency.

Consider the implicit system Σ :

$$\delta^{(1)} * Ex = Ax + Bu + Ex_0\delta, y = Cx, \tag{6}$$

together with, for every pair $(x_0, u) \in R^n \times C^m_{imp}$, the solution set $S(x_0, u)$ (3).

Definition 2.1.

Consider Σ. A point $x_0 \in R^n$ is called *output consistent* if

$$\exists\, u \in C^m_{sm} \exists\, x \in S(x_0, u) : y \in C^r_{sm} \text{ and } y(0^+) = C[x(0^+)] = Cx_o.$$

A point $x_0 \in R^n$ is called *weakly output* consistent if

$$\exists\, u \in C^m_{sm} \exists\, x \in S(x_0, u) : y \in C^r_{sm}.$$

It is obvious that the concepts in Definition 2.1 generalize those in Definition 1.1; the former reduce to the latter if $C = I$, and then we will speak of (weak) *state* consistency. Now, let us denote the spaces of state consistent and weakly state consistent points by $I_s = I_s(\Sigma)$ and $I_s^w = I_s^w(\Sigma)$, respectively. In addition, the spaces of output consistent and weakly output consistent points will be denoted by $I_o = I_o(\Sigma)$ and $I_o^w = I_o^w(\Sigma)$, respectively. Finally, we define

$$\mathcal{W}_f = \mathcal{W}_f(\Sigma) := \{x_0 \in R^n \mid \exists\, x \in S(x_0, 0) \cap \mathcal{C}_{p-imp}^n : y = 0\}; \tag{7}$$

$\mathcal{W}_f$ is the space of points $x_0 \in R^n$ that are *strongly controlled* by *free response* (in [8, Definition 3.1] a point x_0 is called *strongly controllable* if there exists an input $u \in \mathcal{C}_{p-imp}^m$ and a state trajectory $x \in S(x_0, u) \cap \mathcal{C}_{p-imp}^n$ such that $y = 0$). The spaces I_s, I_s^w and $\mathcal{W}_f$ are characterized in [8, Section 3] and all relevant information from [8] is summarized in Propositions 2.2 and 2.4. The spaces I_o and I_o^w then follow from our first main result, Theorem 2.7.

Proposition 2.2.

I_s is the largest subspace $\mathcal{L}$ that satisfies $\mathcal{L} \subset A^{-1}[E(\mathcal{L}) + im(B)]$. In addition, $I_s = A^{-1}[E(I_s) + im(B)]$. Moreover, $I_s^w = I_s + ker(E)$.

Proposition 2.3.

$I_s \supset ker(A)$.

Proof. Assume that $Ax_0 = 0$. Then $x := \delta^{(-1)} * \delta x_0 = Hx_0$ is smooth ($x(t) = x_0$, constant, on R^+), $x(0^+) = x_0$, and $x \in S(x_0, 0)$ since $\delta^{(1)} * Ex = \delta^{(1)} * \delta^{(-1)} * Ex_0 = Ax + Ex_0\delta$.

Proposition 2.4.

$\mathcal{W}_f$ is the smallest subspace $\mathcal{K}$ that satisfies $\mathcal{K} \supset E^{-1}[A(\mathcal{K} \cap ker(C))]$. In addition, $\mathcal{W}_f = E^{-1}[A(\mathcal{W}_f \cap ker(C))]$. The algorithm $\mathcal{W}_0 := ker(E), \mathcal{W}_{i+1} := E^{-1}[A(\mathcal{W}_i \cap ker(C))]$ is such that $\mathcal{W}_0 \subset \mathcal{W}_1 \subset \cdots \subset \mathcal{W}_n = \mathcal{W}_f$.

I_s and $\mathcal{W}_f$ are *dual* concepts [6], [9]. Let the system Σ_1 be represented by $\delta^{(1)} * Ex = Ax + Bu + Ex_0\delta$, and let Σ_1' denote the dual system $\delta^{(1)} * E'w = A'w + E'w_0\delta, z = B'w$. Then $E(I_s(\Sigma_1)) = [\mathcal{W}_f(\Sigma_1')]^\perp$ [8, Theorem 3.12]. Similarly, if Σ_2 denotes the system $\delta^{(1)} * Ex = Ax + Ex_0\delta, y = Cx$, and Σ_2' denotes its dual $\delta^{(1)} * E'w = A'w + C'v + E'w_0\delta$, then $\mathcal{W}_f(\Sigma_2) = [E'(I_s(\Sigma_2'))]^\perp = E^{-1}[I_s(\Sigma_2')]^\perp$.

The *intersection* of I_s and $\mathcal{W}_f$ turns out to equal the space $\mathcal{R}_{sm}^{i/o} = \mathcal{R}_{sm}^{i/o}(\Sigma)$ of points that are *instantaneously reachable from the origin* [6] by *smooth output generating* inputs in $\mathcal{C}_{sm}^m$:

$$\mathcal{R}_{sm}^{i/o} := \{x(0^+) \mid u \in \mathcal{C}_{sm}^m, x \in S(0, u) \text{ are such that } y \in \mathcal{C}_{sm}^r\}. \tag{8}$$

Lemma 2.5 [8, Main Lemma 2.5].

Let $x_0 \in R^n, u = u_1+u_2, u_1 \in \mathcal{C}^m_{p-imp}, u_2 \in \mathcal{C}^m_{sm}, x = x_1+x_2 \in S(x_0,u), x_1 \in \mathcal{C}^n_{p-imp}, x_2 \in \mathcal{C}^n_{sm}$. Then $\delta^{(1)} * Ex_1 + E[x_2(0^+)]\delta = Ax_1 + Bu_1 + Ex_0\delta$ and $\delta^{(1)} * Ex_2 = Ax_2 + Bu_2 + E[x_2(0^+)]\delta$.

Proposition 2.6.

$I_s \cap \mathcal{W}_f = \mathcal{R}^{i/o}_{sm}, I^w_s \cap \mathcal{W}_f = ker(E) + \mathcal{R}^{i/o}_{sm}$.

The algorithm $\mathcal{R}_0 := I_s \cap ker(E), \mathcal{R}_{i+1} := I_s \cap E^{-1}[A(\mathcal{R}_i \cap ker(C))]$ is such that $\mathcal{R}_0 \subset \mathcal{R}_1 \subset \cdots \subset \mathcal{R}_n = \mathcal{R}^{i/o}_{sm}$.

Proof. First statement. Let $u \in \mathcal{C}^m_{sm}, x = x_1 + x_2 \in S(0,u), x_1$ impulsive and x_2 smooth, and $y = Cx \in \mathcal{C}^r_{sm}$. If $x_0 := x(0^+) = x_2(0^+)$, then, by Lemma 2.5, $\delta^{(1)} * E(-x_1) = A(-x_1) + Ex_0\delta, C(-x_1) = 0$, and also $\delta^{(1)} * Ex_2 = Ax_2 + Bu + Ex_0\delta$. Hence, $x_0 \in I_s \cap \mathcal{W}_f$. Conversely, let $x_0 \in I_s \cap \mathcal{W}_f$. Then there exist a control $u \in \mathcal{C}^m_{sm}$ and a state trajectory $x_2 \in S(x_0,u) \cap \mathcal{C}^n_{sm}$ such that $\delta^{(1)} * Ex_2 = Ax_2 + Bu + Ex_0\delta$ and $x_2(0^+) = x_0$. In addition, for some $x_1 \in S(x_0,0) \cap \mathcal{C}^n_{p-imp}, \delta^{(1)} * Ex_1 = Ax_1 + Ex_0\delta$ and $Cx_1 = 0$. Consequently, with $x := -x_1 + x_2, \delta^{(1)} * Ex = Ax + Bu, Cx \in \mathcal{C}^r_{sm}$ and $x(0^+) = x_2(0^+) = x_0$. Thus, $x_0 \in \mathcal{R}^{i/o}_{sm}$. Second statement. By Propositions 2.2, 2.4 and Lemma 1.3, $I^w_s \cap \mathcal{W}_f = [I_s + ker(E)] \cap \mathcal{W}_f = ker(E) + [I_s \cap \mathcal{W}_f] = ker(E) + \mathcal{R}^{i/o}_{sm}$. Third statement. Obviously, by Proposition 2.4 and the foregoing, the algorithm $\mathcal{K}_i := I_s \cap \mathcal{W}_i$ is such that $\mathcal{K}_0 \subset \mathcal{K}_1 \subset \cdots \subset \mathcal{K}_n = \mathcal{R}^{i/o}_{sm}$. We have $\mathcal{R}_0 = \mathcal{K}_0$ and $\mathcal{R}_1 = I_s \cap E^{-1}[A(\mathcal{K}_0 \cap ker(C))] \subset I_s \cap E^{-1}[A(\mathcal{W}_0 \cap ker(C))] = \mathcal{K}_1$. On the other hand, if $x_0 \in \mathcal{K}_1$, i.e., if $x_0 \in I_s$ and $Ex_0 = A\overline{x}$ with $E\overline{x} = 0$ and $C\overline{x} = 0$, then $\overline{x} \in A^{-1}[E(I_s)] \subset I_s$ (Proposition 2.2), and thus $x_0 \in I_s \cap E^{-1}[A(\mathcal{K}_0 \cap ker(C))] = \mathcal{R}_1$. Now, assume that $\mathcal{R}_i = \mathcal{K}_i$. Then, as in the above, $\mathcal{R}_{i+1} \subset \mathcal{K}_{i+1}$. Conversely, if $x_0 \in I_s$ and $Ex_0 = A\overline{x}, \overline{x} \in \mathcal{W}_i$ and $C\overline{x} = 0$, then $\overline{x} \in I_s \cap \mathcal{W}_i \cap ker(C) = \mathcal{K}_i \cap ker(C) = \mathcal{R}_i \cap ker(C)$, and this completes the proof by induction.

Theorem 2.7.

Consider Σ. It holds that $I_o = I_s + [\mathcal{W}_f \cap ker(C)], I^w_o = I_s + \mathcal{W}_f = I^w_s + \mathcal{W}_f$.

Proof. First statement. Obviously, by definition, $\mathcal{W}_f \cap ker(C) \subset I_o$ and $I_s \subset I_o$. Conversely, let $x_0 \in R^n$ be output consistent. Then there exist a smooth input u and a state trajectory $x = x_1 + x_2, x_1$ impulsive and x_2 smooth, such that $y = Cx$ is smooth and $C[x(0^+)] = C[x_2(0^+)] = Cx_0$. Thus, from Lemma 2.5, $x_2(0^+) \in I_s$, but also $\delta^{(1)} * Ex_1 = Ax_1 + E[x_0 - x_2(0^+)]\delta$ and $Cx_1 = 0$ and hence $[x_0 - x_2(0^+)] \in \mathcal{W}_f \cap ker(C)$. Second statement. Every $x_0 \in \mathcal{W}_f$ is clearly weakly output consistent and also $I_s \subset I_o \subset I^w_o$. Conversely, let $x_0 \in R^n$ be weakly output consistent. Then there exist a smooth input u and a state trajectory $x = x_1 + x_2 \in S(x_0, u)$, with x_1 impulsive and x_2 smooth, such that $Cx_1 = 0$. As in the first half of the proof, it follows from Lemma 2.5 that $[x_0 - x_2(0^+)] \in \mathcal{W}_f$ and $x_2(0^+) \in I_s$. This completes the proof.

Note that Theorem 2.7 reduces to the last statement in Proposition 2.2 if $C = I$, since, then, $\mathcal{W}_f = ker(E)$ by (7) or Proposition 2.4.

Corollary 2.8.

Consider Σ and its dual Σ': $\delta^{(1)} * E'w = A'w + C'v + E'w_0\delta, z = B'w$. Then

$$ker(E) + \mathcal{R}^{i/o}_{sm}(\Sigma) = [E'(I^w_o(\Sigma'))]^\perp, E(\mathcal{R}^{i/o}_{sm}(\Sigma)) = [I^w_o(\Sigma')]^\perp,$$

$$E^{-1}[A(\mathcal{R}^{i/o}_{sm}(\Sigma) \cap ker(C))] = [E'(I_o(\Sigma'))]^\perp.$$

Proof. By Theorem 2.7, $E'(I^w_o(\Sigma')) = E'(I_s(\Sigma')) + E'(\mathcal{W}_f(\Sigma'))$. Hence $[E'(I^w_o(\Sigma'))]^\perp = [E'(I_s(\Sigma'))]^\perp \cap [E'(\mathcal{W}_f(\Sigma'))]^\perp = \mathcal{W}_f(\Sigma) \cap E^{-1}[\mathcal{W}_f(\Sigma')]^\perp = \mathcal{W}_f(\Sigma) \cap E^{-1}[E(I_s(\Sigma))] = \mathcal{W}_f(\Sigma) \cap [I_s(\Sigma) + ker(E)] = ker(E) + \mathcal{R}^{i/o}_{sm}(\Sigma)$, by Lemma 1.3 and Proposition 2.6. In addition, by the foregoing, $E[\mathcal{R}^{i/o}_{sm}(\Sigma)] = E[E'(I^w_o(\Sigma'))]^\perp = [(E')^{-1}(E'(I^w_o(\Sigma')))]^\perp = [I^w_o(\Sigma')]^\perp$ (Lemma 1.3). Next, we have $I_o(\Sigma') = I_s(\Sigma') + [\mathcal{W}_f(\Sigma') \cap ker(B')]$ (Theorem 2.7) and thus $E'(I_o(\Sigma')) = E'(I_s(\Sigma')) + E'[\mathcal{W}_f(\Sigma') \cap ker(B')]$.
Hence, $[E'(I_o(\Sigma'))]^\perp = \mathcal{W}_f(\Sigma) \cap E^{-1}[E(I_s(\Sigma)) + im(B)] = E^{-1}[A(\mathcal{W}_f(\Sigma) \cap ker(C))] \cap E^{-1}[E(I_s(\Sigma)) + im(B)]$ (Proposition 2.4) $= E^{-1}\{A(\mathcal{W}_f(\Sigma) \cap ker(C)) \cap [E(I_s(\Sigma)) + im(B)]\} = E^{-1}[A(\mathcal{W}_f(\Sigma) \cap ker(C)) \cap A(I_s(\Sigma))]$ (Proposition 2.2, Lemma 1.3) $= E^{-1}[A(\mathcal{W}_f(\Sigma) \cap I_s(\Sigma) \cap ker(C))]$ (Lemma 1.3, Proposition 2.3) $= E^{-1}[A(\mathcal{R}^{i/o}_{sm}(\Sigma) \cap ker(C))]$ (Proposition 2.6). This completes the proof.

Corollary 2.9.

Every $x_0 \in R^n$ is output consistent if and only if $I_s + [\mathcal{W}_f \cap ker(C)] = R^n$. Every $x_0 \in R^n$ is weakly output consistent if and only if $I_s + \mathcal{W}_f = R^n$.

Now, the conditions in Corollary 2.9 will be *recasted* in terms of R^l instead of R^n. Central instrument for this refashioning operation is Lemma 2.10, which leads to our second main result, Theorem 2.11.

Lemma 2.10.

Let $\mathcal{L} \subset R^n$.

(a) Assume that $\mathcal{L} \subset E^{-1}[A(\mathcal{L}) + im(B)] + ker(A)$. Then $E(I_s) + im(B) + A(\mathcal{L}) = R^l \Leftrightarrow im(E) + im(B) + A(\mathcal{L}) = R^l$.

(b) $I_s + \mathcal{L} = R^n \Leftrightarrow E(I_s) + im(B) + A(\mathcal{L}) \supset im(A)$.

(c) Assume that $[EAB]$ is of full row rank. If $\mathcal{L} \subset E^{-1}[A(\mathcal{L}) + im(B)] + ker(A)$, then $I_s + \mathcal{L} = R^n \Leftrightarrow im(E) + im(B) + A(\mathcal{L}) = R^l$.

Proof. Part (a), $\Rightarrow$: Trivial. Conversely, let Σ' denote the dual system of (2): $\delta^{(1)} * E'w = A'w + E'w_0\delta, z = B'w$. Since $A(\mathcal{L}) \subset A[E^{-1}[A(\mathcal{L}) + im(B)]]$, it follows that $[A(\mathcal{L})]^\perp \supset (A')^{-1}[E'[[A(\mathcal{L})]^\perp \cap ker(B')]]$. According to Proposition 2.4, the algorithm $\mathcal{W}_0 := ker(E'), W_{j+1} := (E')^{-1}[A'(\mathcal{W}_j \cap ker(B'))]$ is such that $\mathcal{W}_0 \subset \mathcal{W}_1 \subset \cdots \subset \mathcal{W}_l = \mathcal{W}_f(\Sigma')$. We will show by induction that, for every $j = 0, 1, \ldots, l$,

$$\mathcal{W}_j \cap ker(B') \cap [A(\mathcal{L})]^\perp = 0, \tag{9}$$

if $\mathcal{W}_0 \cap ker(B') \cap [A(\mathcal{L})]^\perp = 0$. First, let $w_0 \in \mathcal{W}_1, B'w_0 = 0$ and $w_0 \in [A(\mathcal{L})]^\perp$. Then $E'w_0 = A'\bar{w}$ with $B'\bar{w} = 0, E'\bar{w} = 0$ for some $\bar{w} \in R^l$. Thus, also, $\bar{w} \in (A')^{-1}[E'[[A(\mathcal{L})]^\perp \cap ker(B')]] \subset [A(\mathcal{L})]^\perp$, and hence $\bar{w} = 0$. Consequently, $E'w_0 = 0$ and thus $w_0 = 0$. Next, let (9) be true for $j \in \{0, ..., l-1\}$, and let $w_0 \in \mathcal{W}_{j+1}, B'w_0 = 0, w_0 \in [A(\mathcal{L})]^\perp$. Then there exists a $\bar{w} \in \mathcal{W}_j \cap ker(B')$ such that $E'w_0 = A'\bar{w}$. Hence $\bar{w} \in [A(\mathcal{L})]^\perp$ and thus $\bar{w} = 0$ and therefore $w_0 = 0$. We conclude that $\mathcal{W}_f(\Sigma') \cap ker(B') \cap [A(\mathcal{L})]^\perp = 0$ if $ker(E') \cap ker(B') \cap [A(\mathcal{L})]^\perp = 0$. Part (b). By Proposition 2.3, $I_s + \mathcal{L} = R^n$ if and only if $A(I_s) + A(\mathcal{L}) = im(A)$. By Proposition 2.2 and Lemma 1.3, then, $I_s + \mathcal{L} = R^n \Leftrightarrow [E(I_s) + im(B)] \cap im(A) + A(\mathcal{L}) = im(A) \Leftrightarrow [E(I_s) + im(B) + A(\mathcal{L})] \cap im(A) = im(A)$ (Lemma 1.3) $\Leftrightarrow [E(I_s) + im(B) + A(\mathcal{L})] \supset im(A)$. Part (c), $\Leftarrow$: Combine (a) and (b). The converse follows directly from (b) if $im(E) + im(B) + im(A) = R^l$. This completes the proof.

Theorem 2.11.

Consider Σ and assume that $[EAB]$ is of full row rank. Then

$$I_0(\Sigma) = R^n \Leftrightarrow im(E) + im(B) + A[\mathcal{W}_f \cap ker(C)] = R^l, \tag{10}$$

$$I_0^w(\Sigma) = R^n \Leftrightarrow im(E) + im(B) + A[\mathcal{W}_f] = R^l. \tag{11}$$

Proof. It is easily seen that $\mathcal{W}_f \cap ker(C) \subset \mathcal{W}_f = E^{-1}[A(\mathcal{W}_f \cap ker(C))]$ (Proposition 2.4) $\subset E^{-1}[A(\mathcal{W}_f \cap ker(C)) + im(B)] \subset E^{-1}[A(\mathcal{W}_f) + im(B)]$.

Now combine Corollary 2.9 with Lemma 2.10 (c).

Corollary 2.12.

Consider Σ. Then $E(\mathcal{W}_f) \cap E(I_s) = 0 \Leftrightarrow \mathcal{R}_{sm}^{i/0} = ker(E) \cap I_s \Leftrightarrow E(\mathcal{R}_{sm}^{i/0}) = 0$. If $[E', A', C']$ is of full row rank, then $E(\mathcal{W}_f) \cap E(I_s) = 0$ if and only if $ker(E) \cap ker(C) \cap A^{-1}(EI_s) = 0$.

Proof. By Proposition 2.6 and Lemma 1.3, $E(\mathcal{W}_f) \cap E(I_s) = E(\mathcal{W}_f \cap I_s) = E(\mathcal{R}_{sm}^{i/o})$, and $ker(E) \cap I_s \subset \mathcal{R}_{sm}^{i/o} \subset I_s$. This yields the first claim. Next, let Σ' denote the dual system of Σ. Then, from Theorem 2.11, $I_0^w(\Sigma') = R^l$ if and only if $im(E') + im(C') + A'(\mathcal{W}_f(\Sigma')) = R^n$. On the other hand, from Corollary 2.8, $I_0^w(\Sigma') = R^l$ if and only if $E(\mathcal{R}_{sm}^{i/o}) = 0$. Combination of these statements with the above proves the second claim.

Example.

Observe Σ: $\delta^1 * \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_{01} \\ x_{02} \end{bmatrix} \delta$,

$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$. Directly, $x_1 = 0, x_2 + u + x_{01}\delta = 0$. For every x_{02} the smooth input $u = -Hx_{02}$ yields, with $x_{01} = 0, x_2 = Hx_{02}, x_2(0^+) = x_{02}$ and hence $I_s = ker(E)$. In addition, $x_2 = -x_{01}\delta$ is such that $\begin{bmatrix} 0 \\ x_2 \end{bmatrix} \in S(x_0, 0), y = 0$ for every $x_0 = \begin{bmatrix} x_{01} \\ x_{02} \end{bmatrix} \in R^2$. Thus, $\mathcal{W}_f = R^2$. However, $\mathcal{W}_f \cap ker(C) = ker(E)$ and thus $I_0 \neq R^2, I_o^w = R^2$. Note that $I_s = I_s^w \neq R^2$! Also, $\mathcal{R}_{sm}^{i/o} = I_s \cap ker(E)$; note that the dual of Σ is Σ itself!

Remarks.

1. Observe that I_s is *not* appearing in (10) - (11), and that Theorem 2.11 covers Proposition 1.2.
2. Combination of Proposition 2.6 with Corollary 2.8 yields $\mathcal{R}_{sm}^{i/o}(\Sigma) = I_s(\Sigma) \cap [E'(I_o(\Sigma'))]^\perp$. Thus, $\mathcal{R}_{sm}^{i/o}(\Sigma) = I_s(\Sigma) \cap ker(E)$ if $I_o(\Sigma') = R^l$. The converse is not true, see the Example.
3. By Propositions 2.2 - 2.4, 2.6 and Lemma 1.3, $E(\mathcal{W}_f) \cap A(I_s) = E(\mathcal{W}_f) \cap [E(I_s) + im(B)] \cap im(A) = E(\mathcal{W}_f) \cap [E(I_s) + im(B)]$, but also $E(\mathcal{W}_f) \cap A(I_s) = A[\mathcal{W}_f \cap ker(C)] \cap A(I_s) \cap im(E) = A(\mathcal{R}_{sm}^{i/o} \cap ker(C)) \cap im(E)$. Thus, if $ker(E) \cap ker(C) \cap I_s = 0$, then $E(\mathcal{W}_f) \cap [E(I_s) + im(B)] = 0$, by Corollary 2.12. If, moreover, $I_o^w = R^n$, then $B^{-1}[im(E)] = B^{-1}[E(I_s)]$, even if $im(E) \neq E(I_s)$.
4. If $I_{imp} = \{x_0 \in R^n \mid \exists\ u \in C_{p-imp}^m : S(x_0, u) \cap C_{p-imp}^n \neq 0\}$, then

$\mathcal{W}_f \subset I_{imp} = E^{-1}[A(I_{imp}) + im(B)]$ [8, Section 3]. Consequently, by Lemma 2.10 (c), [1, Corollary 3.6] and [8, Theorem 3.2], $I_s + I_{imp} = R^n \Leftrightarrow im(E) + im(B) + A(I_{imp}) = R^n \Leftrightarrow [sE - A, -B]$ is right invertible as a rational matrix, provided that $[EAB]$ is of full row rank.

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